ADSL/VDSL Principles
- 3 -
Loop Analysis

In this chapter:

- A review of ABCD parameters
- Discussion of bridged taps
- Application of ABCD parameters to twisted pair characteristics
- Discussion of ADSL and VDSL test loops

This chapter presents methods to analyze transmission parameters of twisted pairs. This chapter provides a generic procedure using ABCD parameters that is applicable to almost every type of loop. Understanding the properties of twisted pairs is necessary for understanding many aspects of ADSL and VDSL modulation methods. Being able to characterize different types of loops is also necessary for benchmarking the limits of performance for a modulation. In this spirit, a discussion is given on the various test loops to be used when characterizing theoretical performance achievable on a twisted pair, as well as actual performance of different modulation schemes. Various standards bodies have defined these loops to benchmark proposed modulation schemes and define the performance goals of DSL technologies.\(^1\),\(^2\) In addition, several sections deal with the definition and use of ABCD parameters, providing the tools to characterize virtually any loop configuration.

**ABCD Parameters**

Before discussing various test loops, a review of ABCD transmission parameters will be helpful. ABCD parameters are useful in characterizing two-port networks. A two-port network is generically defined to have a single unique input port and a single unique output port with some transfer function between the two. These parameters mathematically relate the input voltage and current to the output voltage and current. Figure 3.1 shows a generic two-port network. In this figure, the output voltage and current \(V_2\) and \(I_2\) are related to the input voltage and current \(V_1\) and \(I_1\) in terms of ABCD parameters as in Eqtn. 3.1.

**FIGURE 3.1** A generic two-port network.

**Eqtn. 3.1**

Note that Eqtn. 3.1 characterizes the network without making any assumptions about the network’s termination. The parameters A, B, C, and D are specific to the transfer function of the network.
Figure 3.2 shows two networks in tandem. Each network is independently characterized by its own 
ABCD parameters (denoted by subscript 1 and subscript 2).

**FIGURE 3.2 Two two-port networks in tandem; each network has unique ABCD parameters.**

The effective ABCD parameters for the entire system can be given by a simple matrix multiplication of the individual parameters such that Eqtn. 3.2 is true. ³

**Eqtn. 3.2**

If another two-port network was added to that in Figure 3.2 to the right of the existing networks, the new equivalent ABCD parameters of the entire system could be found by matrix multiplication of the ABCD matrix for the new network by the result of Eqtn. 3.2. Thus, if the parameters of each individual network are given in ABCD form, it is a trivial task to find the characteristics of the network as a whole.

ABCD parameters are useful for characterizing twisted-pair loops because not all loops consist of a uniform gauge and a perfect termination. For example, a loop might consist of 9 kft of #26 gauge cable followed by 2 kft of #24 gauge cable. The insertion loss analysis in Chapter 2 did not account for such a case.

**ABCD Parameters of a Lumped Impedance**

One of the simplest cases of ABCD parameters is for a lumped impedance as shown in Figure 3.3.

**FIGURE 3.3 A two-port network consisting of a lumped parallel impedance.**

In this case, \( V_2 = V_1 \) independent of \( I_2 \). Thus, the ABCD parameter equations can be written as in Eqtn. 3.3.

**Eqtn. 3.3**

Solving the first equation is trivial and yields Eqtn. 3.4.

**Eqtn. 3.4**

Eqtn. 3.3 is best solved using Kirkoff’s current law at point N. This results in Eqtn. 3.5.

**Eqtn. 3.5**

Comparing Eqtn. 3.5 and Eqtn. 3.3, it should be clear that Eqtn. 3.6 is true.

**Eqtn. 3.6**

Thus, the ABCD matrix for a two-port network consisting only of a shunt impedance is given by Eqtn. 3.7.
Eqtn. 3.7

Z in Eqtn. 3.7 represents the shunt impedance. Note that Z may be complex because no restriction is placed on it to be real.

**ABCD Parameters of a Uniform Twisted Pair**

For a uniform twisted pair, finding the ABCD parameters is a bit more involved than for a simple lumped element. Chapter 2 showed that a wave traveling on a twisted pair is attenuated exponentially with distance and that the exponential depends on the pair’s propagation constant. The voltage at a point on the line is the sum of a forward traveling wave at that point and the backward traveling wave. Figure 3.4 shows a twisted pair with the forward and backward traveling waves labeled at the end of the pair (x=L). Eqtn. 3.8 gives the voltage at the other end of the twisted pair (x=0).

**FIGURE 3.4 A twisted pair transmission line; the forward (V+) and backward (V-) traveling waves are labeled.**

Eqtn. 3.8

The reflection coefficient in Eqtn. 3.8 was given in Eqtn. 2.10 and is repeated in Eqtn. 3.9.

Eqtn. 3.9

Note that in Eqtn. 3.8, the voltage at x=0 is the sum of a forward and backward traveling wave seen at x=L. The forward traveling wave will be larger at x=0 because it propagates from x=0 to x=L (indicated by the positive exponent in Eqtn. 3.8). The backward traveling wave will be larger at x=L because it propagates from x=L to x=0 (indicated by the negative exponential exponent in Eqtn. 3.8).

For ABCD parameters, it is helpful to write Eqtn. 3.8 in terms of the two-port output voltage and current rather than in terms of the positive traveling wave voltage, \( V^+_{x=L} \). Figure 3.5 shows the equivalent two-port network representation of the uniform twisted pair.

**FIGURE 3.5 An equivalent two-port network for a twisted pair.**

Dependence of all values on frequency has been dropped for notational simplicity. Because the output voltage is the sum of forward and backward wave voltages at x=L, you can immediately find the expression for \( V_2 \) given in Eqtn. 3.10.

Eqtn. 3.10

Solving for \( V^+_{x=L} \) is then relatively straightforward as shown in Eqtn. 3.11.

Eqtn. 3.11

To find \( V_1 \), Eqtn. 3.11 can be substituted into Eqtn. 3.8 and upon simplifying, results in Eqtn. 3.12.
Eqtn. 3.12
To further simplify Eqtn. 3.12, Eqtn. 3.9, the definition of reflection coefficient, is used. After substituting and simplifying, Eqtn. 3.12 can be rewritten as Eqtn. 3.13.

Eqtn. 3.13
Recall that $Z_0$ is the characteristic impedance of the transmission line at $x=L$, and $Z_L$ is the load impedance. The second line in Eqtn. 3.13 results from simply using Ohm’s law and the third line from combining like terms. Note that the definitions of hyperbolic cosine (cosh) and hyperbolic sine (sinh) are present in the last line of Eqtn. 3.13. Substituting for these yields Eqtn. 3.14.

Eqtn. 3.14
Similarly, the second two ABCD parameters can be found (the details are left to you in Exercise 2 at the end of the chapter) as in Eqtn. 3.15.

Eqtn. 3.15
Thus, the ABCD parameters for a uniform twisted pair (or any generic uniform transmission line) are as shown in Eqtn. 3.16.

Eqtn. 3.16

Bridged Taps

Sometimes, a twisted pair running to a subscriber might have another, unused twisted pair section connected at some point along its length. The other end of the unused section is left open circuited (see Figure 3.6). The unused section of twisted pair is called a bridged tap.

FIGURE 3.6 A common bridged tap configuration on a twisted pair.

The purpose of a bridged tap is usually to leave flexibility in the location of a subscriber using the loop. This flexibility is necessary when twisted pairs are installed prior to being used because in many cases the actual location of the subscriber is not yet known (for example, when cables are laid in a neighborhood under development). A bridged tap causes reflections at the open circuit end producing dips in the transfer function of the loop to which it is attached. Note that the propagation constant and characteristic impedance of the bridged tap might or might not be the same as the loop to which the tap is connected. Also note that the main loop itself might have different line characteristics on each side of the bridged tap.

The ABCD parameters of a bridged tap are the same as any other uniform twisted pair as given in Eqtn. 3.16. In the bridged tap case, however, you do not need to be concerned with the transfer function of the tap (because the signal does not propagate along the bridged tap) but rather the effect that the tap’s presence has on the main loop’s transfer function. Using Eqtn. 3.16, a bridged tap’s ABCD equations are shown in Eqtn. 3.17.
Eqtn. 3.17

The simplification in both equations comes because the bridged tap, by definition, is open circuited at one end, and thus, \( I_2 = 0 \).

Dividing the two equations in Eqtn. 3.17 yields Eqtn. 3.18.

Eqtn. 3.18

In Eqtn. 3.18, the hyperbolic cotangent (coth) is used. From the main loop’s point of view, the effect of the bridged tap would be the same as the effect of a lumped impedance across the loop with a value of \( Z_{\text{intap}} \). The ABCD parameters of a lumped impedance were derived in Eqtn. 3.7. Substituting the effective shunt impedance of a bridged tap from Eqtn. 3.17 into Eqtn. 3.5 yields Eqtn. 3.19.

Eqtn. 3.19

These ABCD parameters do not describe the parameters along the length of the bridged tap but rather the effect of the bridged tap on the main loop. The following section, "Loop Analysis," illustrates how this information can be used to model a loop with any structure.

**Loop Analysis**

At this point, you can analyze a wide variety of loops by finding the ABCD parameters of each section of the loop and performing matrix multiplications. For example, consider the loop shown in Figure 3.7.

**FIGURE 3.7** An example twisted pair to illustrate the usefulness of ABCD parameters.

This loop consists of 9 kft of #26 gauge cable followed by a total of 3 kft of #24 gauge cable. Because two bridge taps exist on the #24 section (each 1.5 kft of #26 gauge cable), the #24 section is broken into one 2 kft section and two 500 foot sections. The ABCD parameters for the entire cable are found by multiplying the ABCD parameters for the individual sections along the loop. This is shown generically in Eqtn. 3.20.

Eqtn. 3.20

The values shown in each set of brackets represent the ABCD parameters of the listed section. Note that six sets of ABCD parameters are necessary to analyze this loop as labeled in the figure and shown in Eqtn. 3.20. This loop is T1.601 #9 and is analyzed in detail later in the chapter.

The ABCD parameters for each term in Eqtn. 3.20 were discussed in the preceding sections. The information necessary to find the ABCD parameters includes the gauge and length of each section (along with whether the section is part of the main loop or a bridged tap). A cable of this type, with varying gauge and multiple bridged taps, illustrates the usefulness of ABCD parameters.
Input Impedance, Transfer Functions, and Insertion Loss

Given that you can characterize any twisted pair in terms of ABCD parameters, you can see how to find three important characteristics about a loop, mainly the input impedance, transfer function, and insertion loss. In each case, an assumption is made that the loop’s ABCD parameters have been found.

For analysis of a loop, specific information beyond the ABCD parameters of the loop is necessary. This information includes the impedance of the source providing a signal to the loop and the impedance of the loop termination. These are sometimes referred to as source impedance (or generator impedance) and termination impedance (or load impedance). Figure 3.8 shows a generic end-to-end loop model.

FIGURE 3.8 A generic end-to-end model of a loop including the generator impedance at the modulator and the load or terminating impedance at the receiver.

$Z_g$ is the impedance of the generator, and $Z_L$ is the load impedance (this analysis assumes that both are 100 ohms). Note that in ADSL and VDSL, $Z_g$ would normally be part of the transmitter design and $Z_L$ part of the receiver design. The input impedance will be a function of the impedance of the loop and of the load (but not $Z_g$). The transfer function will be from $V_1$ to $V_2$. The insertion loss will be a function of the loop, the load impedance, and the generator impedance.

The input impedance of the loop is simple to derive. Given the ABCD parameter equations as shown in Eqtn. 3.1, the input impedance is found by simply dividing the first equation by the second. Regarding Figure 3.8, by Ohm’s law, it follows that $V_2 = I_2 Z_L$. Thus, the input impedance can be found as in Eqtn. 3.21.

**Eqtn. 3.21**

The transfer function of a loop can be found from the first ABCD equation. Once again, using Ohm’s law, the equation can be written as in Eqtn. 3.22.

**Eqtn. 3.22**

Ohm’s law was used to replace $I_2$ by $V_2/Z_L$.

Eqtn. 3.22 can be solved algebraically for the transfer function, $V_2/V_1$ yielding the simple expression in Eqtn. 3.23.

**Eqtn. 3.23**

The insertion loss of the loop is the ratio of power delivered to the load to the power that would have been delivered to the load without the loop present. Essentially, this is the loss caused by inserting the loop between the source and termination. Insertion loss is normally expressed in dB.
If no loop were present ($Z_G$ was connected directly to $Z_L$), the voltage across the load could be found by a simple voltage divider as shown in Eqtn. 3.24.

Eqtn. 3.24

The power delivered to the load in this case would then follow simply as in Eqtn. 3.25.

Eqtn. 3.25

When the loop is present, the voltage $V_1$ can be written as in Eqtn. 3.26.

Eqtn. 3.26

The last part of this equation follows from simple circuit theory (taking into account the voltage drop across $Z_G$). Substituting in the second generic ABCD equation for $I_1$ then yields Eqtn. 3.27.

Eqtn. 3.27

The entire equation can be written in terms of voltages by substituting $V_2 / Z_L$ in for the two $I_2$ terms. This is done in Eqtn. 3.28.

Eqtn. 3.28

Finally, algebraic manipulation can be used to solve Eqtn. 3.28 for $V_2$. The result is shown in Eqtn. 3.29.

Eqtn. 3.29

Now that $V_2$ is known, the power delivered to the load with the loop present can be found as shown in Eqtn. 3.30.

Eqtn. 3.30

The insertion loss of the loop is then defined by Eqtn. 3.31.

Eqtn. 3.31

The results of Eqtn. 3.25 and Eqtn. 3.30 have been used in Eqtn. 3.31 for the power expressions. The insertion loss expression in Eqtn. 3.31 can be algebraically simplified and results in Eqtn. 3.32.

Eqtn. 3.32

This expression for insertion loss will be used extensively later in the book to predict the performance of ADSL and VDSL systems on various loops. For more discussion on ABCD parameters, Appendices C, D, and E of the *Subscriber Loop Signalling and Transmission Handbook* are recommended.4
Basic Loop Configurations

To set requirements, compare different modulation proposals, and demonstrate compliance, standards bodies have defined fixed loops for ADSL and VDSL. (VDSL loops are still being defined. Discussed here are what have been agreed on at the present time.) The following sections present these loops and show the input impedance, transfer function, and insertion loss of each. A generator impedance and load impedance of 100 ohms is assumed throughout.

Note: A null loop, often called CSA #0, which is less than 10 feet of #26 gauge cable, is not discussed. It is sometimes used to show basic operational capability.

ADSL Loops

Figure 3.9 shows the loops used for ADSL. The variety of different loop topologies and gauges lends weight to the value of the previous discussions on ABCD parameters. For these loops, only #24 and #26 gauge cables are used.

The numerical models for sections of #24 and #26 gauge twisted pairs were presented in Chapter 2, "Twisted Pair Environment," and will be used when analyzing the loops. Specifically, the characteristic impedances and propagation constants over frequency of each cable will be necessary for the analysis. In addition, the basic concepts of ABCD parameters are needed along with the input impedance, transfer function, and insertion loss results derived in Eqtn. 3.21, Eqtn. 3.23, and Eqtn. 3.32, respectively.

Figures 3.10-3.17 show graphs of impedance over frequency as well as transfer function and insertion loss for all ADSL loops (except the null loop). The responses are shown up to around 1.2 MHz encompassing the entire ADSL band. For the input impedance and insertion loss graphs, a source and load impedance of 100 ohms is assumed. A few notes about the graphs follow Figure 3.9.

FIGURE 3.9 Test loops specified for ADSL.

- The real part of the input impedance for all loops converges to around 100 ohms, and the imaginary part to zero as frequency increases.

- The insertion loss for the lines with no bridged taps is monotonically decreasing with frequency. The loops with bridged taps have an insertion loss exhibiting more fluctuation. T1.601 Loop #9 is a good example of the effect that bridged taps can have on insertion loss.

- The transfer function (graphed on a log scale) very much follows the shape of the insertion loss. The insertion loss is dependent on the choice of generator impedance (100 ohms in our case), whereas the transfer function is independent of this impedance. Insertion loss and transfer function curves are notably different when the input impedance strays from 100 ohms. Such behavior can be seen in the CSA Loop #4 graphs.

FIGURE 3.10 Input impedance, transfer function magnitude, and insertion loss for T1.601 Loop #7.
FIGURE 3.11 Input impedance, transfer function magnitude, and insertion loss for T1.601 Loop #9.

FIGURE 3.12 Input impedance, transfer function magnitude, and insertion loss for T1.601 Loop #13.

FIGURE 3.13 Input impedance, transfer function magnitude, and insertion loss for CSA Loop #4.

FIGURE 3.14 Input impedance, transfer function magnitude, and insertion loss for CSA Loop #6.

FIGURE 3.15 Input impedance, transfer function magnitude, and insertion loss for CSA Loop #7.

FIGURE 3.16 Input impedance, transfer function magnitude, and insertion loss for CSA Loop #8.

FIGURE 3.17 Input impedance, transfer function magnitude, and insertion loss for the Mid CSA Loop.

VDSL Loops

Figure 3.18 shows the loops currently designated for VDSL. For these three loops, two new types of cables are shown, called DW10 and FP (DW10 stands for Drop Wire #10). DW10 consists of 0.5 mm untwisted conductors insulated with PVC. FP represents untwisted Flat-Pair wires. Short sections of these wires are often used to connect twisted pairs to homes and buildings. These wires are not included in the ADSL models because their effect is negligible.

FIGURE 3.18 Test loops specified for VDSL.

For VDSL, these wires are not negligible because VDSL loops are short, and the frequency range of VDSL signals is higher. Table 3.1 provides the numerical parameter models for DW10 and FP wires.

TABLE 3.1 Parameters for the Numeric Twisted Pair Model for DW10 and Flat-Pair Cable

<table>
<thead>
<tr>
<th>Parameter</th>
<th>DW10</th>
<th>FP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_{oc}$ (ohms/km)</td>
<td>180.93</td>
<td>41.6</td>
</tr>
<tr>
<td>$r_{os}$ (ohms/km)</td>
<td>[infinity]</td>
<td>[infinity]</td>
</tr>
<tr>
<td>$a_c$ (ohms$^4$/km$^4$Hz$^2$)</td>
<td>0.0497223</td>
<td>0.001218</td>
</tr>
<tr>
<td>$a_s$ (ohms$^4$/km$^4$Hz$^2$)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$l_0$ (H/km)</td>
<td>728.87*10$^{-6}$</td>
<td>1000*10$^{-6}$</td>
</tr>
<tr>
<td>$l_{[infinity]}$ (H/km)</td>
<td>543.43*10$^{-6}$</td>
<td>911*10$^{-6}$</td>
</tr>
<tr>
<td>$f_m$ (Hz)</td>
<td>718888</td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>0.75577086</td>
<td>1.195</td>
</tr>
<tr>
<td>$g_o$ (Siemen/Hz*km)</td>
<td>89*10[-9]</td>
<td>53*10[-9]</td>
</tr>
</tbody>
</table>
VDSL Loops #5, #6, and #7 have more than one length defined for them as shown in Figure 3.18. Essentially, the different lengths represent a short, medium, and long length loop for VDSL testing. Note that VDSL Loop #5 and VDSL Loop #6 are similar except for the short section at the end, essentially exercising the different types of end drops. VDSL Loop #7 is meant to test operation in the presence of bridged taps. The other test loops are configurations representing common loop topologies.

Figures 3.19-3.22 show graphs for the selected VDSL loops. Again, for input impedance and insertion loss graphs, the source and load impedances are assumed to be 100 ohms. The graphs are shown up to around 15 MHz, which encompasses most of the usable VDSL band. In some cases (Figure 3.20 and Figure 3.22), more than one loop is shown on a graph to draw a comparison between loops. For loops with a selectable length, the longest length is shown. The following list provides a few notes about the VDSL loop graphs:

- VDSL Loop #2 and VDSL Loop #3 have very similar impedance characteristics, but VDSL Loop #3 is a bit more lossy. This suggests that DW10 is more lossy than FP cable.

- The input impedance of VDSL Loop #6 and VDSL Loop #7 is almost identical. VDSL Loop #7 has more loss because the middle section of #24 gauge twisted pair is almost four times as long as that for VDSL Loop #6.

- Properties similar to the ADSL loops are observed. The real and imaginary parts of the impedances converge similarly. Also, the insertion loss graphs generally decrease with frequency, although they do contain some ripple due to the presence of bridged taps.

**FIGURE 3.19** Input impedance, transfer function magnitude, and insertion loss for the VDSL Loop #1.

**FIGURE 3.20** Input impedance, transfer function magnitude, and insertion loss for the VDSL Loop #2 and VDSL Loop #3.

**FIGURE 3.21** Input impedance, transfer function magnitude, and insertion loss for the VDSL Loop #4.

**FIGURE 3.22** Input impedance, transfer function magnitude, and insertion loss for the VDSL Loop #6 and VDSL Loop #7.

**Summary**

This chapter described a method to analyze loops of many different types using ABCD parameters. Detailed derivations of ABCD parameters for transmission lines were presented, and expressions for
input impedance, insertion loss, and transfer function were derived. Test loops used to benchmark ADSL systems and VDSL systems were presented, and ABCD parameters used to examine their characteristics were described in detail. These results are used in later chapters to derive the theoretical performance limits of ADSL and VDSL.

**Exercises**

1. Prove the relationship given in Eqtn. 3.2 by writing out ABCD equations for each of the networks and solving them for the total network output.

2. Prove the relationship given in Eqtn. 3.15.

3. Simplify Eqtn. 3.14 for the special case where \( Z_o = Z_L \).

4. Find the input impedance, transfer function magnitude, and insertion loss of a 9 kft, #26 gauge cable and a 9 kft, #26 gauge cable with a 2 kft, #26 gauge bridged tap located at 8 kft.

5. Find the transfer function phase response of CSA Loop #6. Why does the slope of the phase increase with increasing frequency?

6. Find the input impedance of CSA Loop #7 in the reverse direction (looking in from the subscriber’s location). Assume a termination at the other end of 100 ohms. How does the input impedance behave when the bridged tap’s length is varied? How does this compare with the input impedance for this loop looking in from the central office?

7. Consider a source generating a signal occupying the band from 0 Hz to 1.0 MHz with a flat power spectral density of \( \text{-}40 \text{dBm/Hz} \). What is the total output power of the source? Compare the total power of the received signal if such a source were the transmitter on each of the ADSL loops. Compare the total power received in only the 0 Hz to 500 kHz range.

8. Consider a source generating a signal occupying the band from 0 Hz to 10 MHz with a flat power spectral density of \( \text{-}60 \text{dBm/Hz} \). What is the total output power of the source? Compare the total power of the received signal if such a source were the transmitter on each of the VDSL loops. Compare the total power received in only the 0 Hz-500 MHz range.

**Endnotes**


